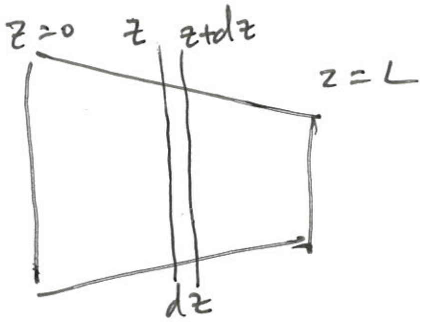
① For 1 dimension, $D = \frac{\Gamma a^2}{2}$

2 dimension, $D = \frac{\Gamma a^2}{4}$

3 dimension $D = \frac{\Gamma a^2}{6}$

As we go from 1 to 2 to 3 dimension, the random walk has more options to jump or hop in various dimension or direction. In a way, the degree of freedom of hopping increases. As a result, the net displacement will decrease. This leads to a lower diffusitivity for higher dimension.

② ⓐ The system is between the two boundary conditions and where we are interested in calculating the concentration profile.

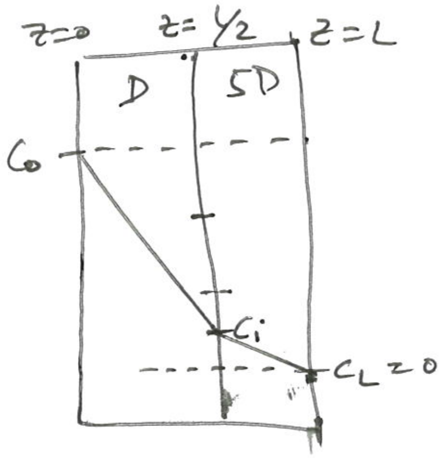


In this case, the system is entire film from

$z=0$ to $z=L$

ⓑ The volumetric element would be the space where the concentration is constant. In this film, it will be the vertical line confined space marked in part ⓐ

③ The flux at steady-state is constant.



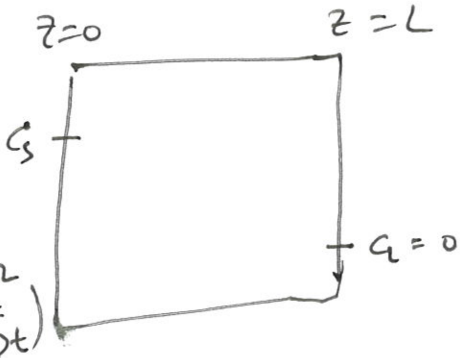
$$\Rightarrow \left[\frac{C_0 - C_i}{(L/2)}\right] D = \left(\frac{C_i - C_L}{L/2}\right) 5D$$

Where C_i is the intermediate concentrate at $z = L/2$

$$\Rightarrow (C_0 - C_i) = 5(C_i - C_L) = 5C_i, \quad C_L = 0$$

$$\Rightarrow C_0 = 6C_i \Rightarrow C_i = C_0/6$$

A rough indication is provided above.



⑭ ⓐ Flux in the case of transient diffusion is proportional to $\sqrt{\frac{D}{\pi t}} \exp\left(\frac{-z^2}{4Dt}\right)$

At $t=0$, flux $= \infty$ at $z = 0$

ⓑ Ratio of flux at two locations at any given time $= \dfrac{\exp\left(-\frac{z_1^2}{4Dt}\right)}{\exp\left(-\frac{z_2^2}{4Dt}\right)}$

$$\Rightarrow \frac{J_1}{J_2} = \exp\left(\frac{z_2^2 - z_1^2}{4Dt}\right) = \exp\left(\frac{z_2^2}{4Dt}\right) \quad \text{as } z_1 = 0$$

$$\frac{J_1}{J_2} = \exp\left(\frac{z_2^2}{4Dt}\right) \quad \text{where} \quad \begin{aligned} z_2 &= 1\,\text{cm} \\ D &= 0.01\ \text{cm}^2/\text{s} \\ t &= 1\ \text{s} \end{aligned}$$

$$\Rightarrow \frac{J_1}{J_2} = \exp\left(\frac{0.01 \times 0.01}{4 \times 0.01 \times 10^{-4} \times 1}\right)$$

$$= \exp\left(\frac{100}{4}\right) = \exp(25) = 7.2 \times 10^{10}$$

This means that compared to flux at $z=0$, flux at $z=1$ cm is almost negligible.

ⓒ To calculate the total amount of gas diffusing inside the polymer, we would need to integrate the flux at $z=0$ with respect to time.

Flux at $z=0$ is $J_{z=0} = -\sqrt{\frac{D}{\pi t}}\,(C_0 - C_s)\exp\left(\frac{-z^2}{4Dt}\right)$

Where $z=0$

Total amount of gas

$$= \int_{t=0}^{t=10s} \text{Flux} \cdot A\, dt$$

$$=$$

$$= A\int_0^{10s} \sqrt{\frac{D}{\pi t}}\, C_s\, dt = C_s A \sqrt{\frac{D}{\pi}} \int_0^{10s} \frac{dt}{\sqrt{t}}$$

⑤ Generally $n_1 = c_1 v_1$

$n_2 = c_2 v_2$

$n = n_1 + n_2 = c_1 v_1 + c_2 v_2$

For gases, total concentration $c = c_1 + c_2$

$= y_1 c + y_2 c$

$\Rightarrow n = y_1 c v_1 + y_2 c v_2$

$\Rightarrow c(y_1 v_1 + y_2 v_2)$

however, $y_1 v_1 + y_2 v_2 = v =$ average velocity of moles

$\Rightarrow n = cv$

⑥ Convective Flux = Average velocity × concentration

= (100 km/h) × (water concentration)

1 liter of water has $\frac{1000}{18}$ moles of water

$\Rightarrow$ Concentration of water = $55.5 \times 10^3 \frac{\text{mol}}{\text{m}^3}$

$\Rightarrow$ Flux = $100 \times \frac{10^3}{3600}$ m/s $\times$ $55.5 \times 10^3 \frac{\text{mol}}{\text{m}^3}$

$\Rightarrow$ flux = $1.54 \times 10^6 \frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$

⑦ For spherical particles which are diffusing in water

$$D = \frac{KT}{f}$$ when particle size >> size of water molecules

⇒ Ration of Diffusion coefficient

$$\frac{D_1}{D_2} = \frac{f_2}{f_1} = \frac{r_2}{r_1}$$ where r = radius of the particle

$$\Rightarrow \frac{D_1}{D_2} = \frac{5}{10} = 1/2$$

⑧ ⓐ $D_{He-Ar} = D_{Ar-He} = 0.001 \ cm^2/s$

ⓑ $D_2 P_2 = D_1 P_1$

$$\Rightarrow D_2 = D_1 \frac{P_1}{P_2} = 0.001 \left(\frac{1}{10}\right) = 10^{-4} \ cm^2/s$$

ⓒ $D \propto T^{3/2}$

$$\frac{D_1}{T_1^{3/2}} = \frac{D_2}{T_2^{3/2}}$$

$$\Rightarrow \frac{D_1}{D_2} = \left(\frac{T_1}{T_2}\right)^{3/2}$$

$$\Rightarrow D_2 = D_1 \left(\frac{T_2}{T_1}\right)^{3/2} = 0.001\left(\frac{373}{298}\right)^{1.5}$$

$$= 0.001 \times 1.4 = 0.0014$$

⑨ Flux is proportional to D

$$D_{vis} = \frac{d^2 P}{32\eta} , \quad D_{knudsen} = \sqrt{\frac{8 k_B T d^2}{9\pi m}}$$

$$D_{molecular} = \frac{8}{3}\sqrt{\frac{2}{m}}\left(\frac{k_B T}{\pi}\right)^{3/2}\frac{1}{P\sigma^2}$$

Note:- exact formula is not important, the aim is to remember that

$D_{vis} \propto P$	$D_{Knudsen} \propto d$	$D_{molecular} \propto \frac{1}{P}$
$D_{vis} \propto d^2$	Independent of pressure	Independent of pore diameter

⇒ We have following answer,

a) Flux dominated by Knudsen diffusion

ⓑ " " " Viscous diffusion

ⓒ " " " Molecular diffusion

ⓓ " " " Molecular diffusion

ⓔ " " " Knudsen diffusion

ⓕ " " " Viscous diffusion.